



Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 Solve the equation $4 \sin \theta + \tan \theta = 0$ for $0^\circ < \theta < 180^\circ$.

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- 2 (a) Find the first three terms in the expansion, in ascending powers of x , of $(2 + 3x)^4$. [2]

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- (b) Find the first three terms in the expansion, in ascending powers of x , of $(1 - 2x)^5$. [2]

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- (c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

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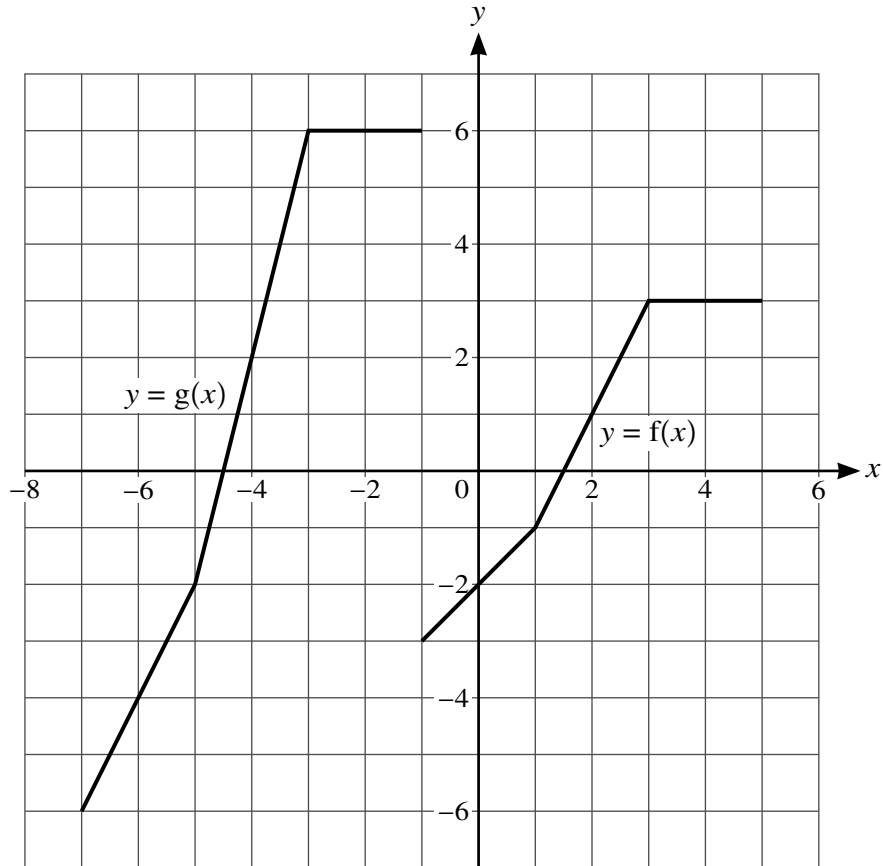
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The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

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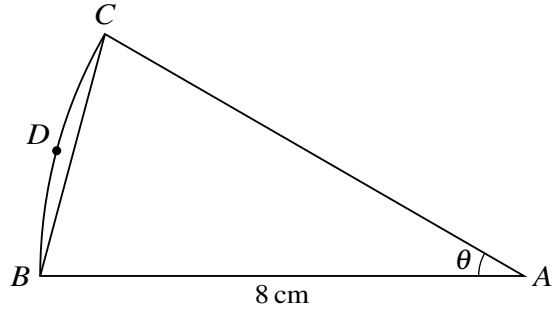
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The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi\text{cm}^2$. The point D lies on the arc BC .

Find the perimeter of the segment BCD . [4]

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- 5 The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

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6 The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p .

(a) Given that the common difference of the progression is not zero, find the value of p . [3]

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(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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7 A curve has equation $y = 2 + 3 \sin \frac{1}{2}x$ for $0 \leq x \leq 4\pi$.

(a) State greatest and least values of y . [2]

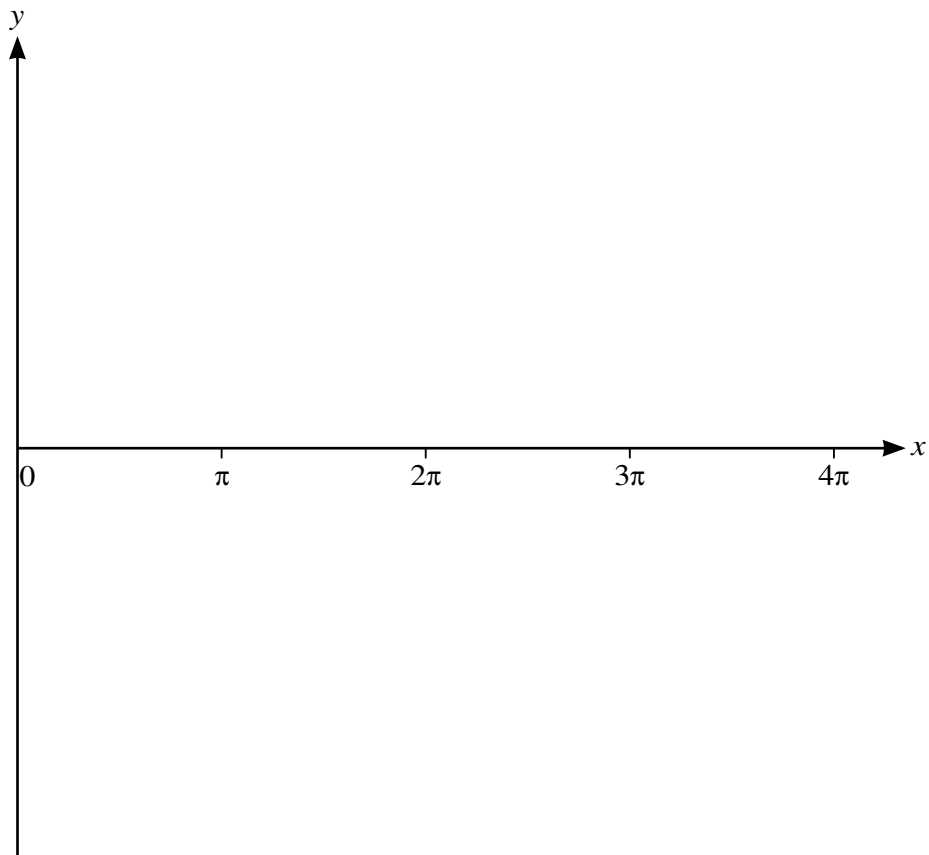
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(b) Sketch the curve. [2]



(c) State the number of solutions of the equation

$$2 + 3 \sin \frac{1}{2}x = 5 - 2x$$

for $0 \leq x \leq 4\pi$. [1]

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8 The functions f and g are defined as follows, where a and b are constants.

$$f(x) = 1 + \frac{2a}{x - a} \text{ for } x > a$$

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b . [4]

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For the rest of this question, you should use the value of a which you found in (a).

(b) Find the domain of f^{-1} . [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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9 Water is poured into a tank at a constant rate of 500 cm^3 per second. The depth of water in the tank, t seconds after filling starts, is $h \text{ cm}$. When the depth of water in the tank is $h \text{ cm}$, the volume, $V \text{ cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25 + h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10 \text{ cm}$. [3]

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(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant.

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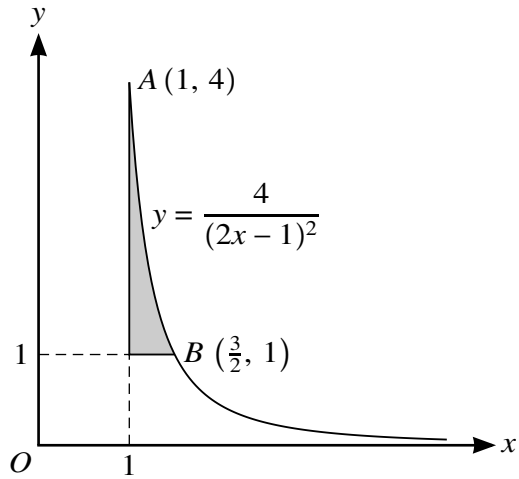
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The diagram shows part of the curve with equation $y = \frac{4}{(2x - 1)^2}$ and parts of the lines $x = 1$ and $y = 1$.
The curve passes through the points $A(1, 4)$ and $B(\frac{3}{2}, 1)$.

- (a) Find the exact volume generated when the shaded region is rotated through 360° about the x -axis. [5]

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- (b) A triangle is formed from the tangent to the curve at B , the normal to the curve at B and the x -axis.

Find the area of this triangle.

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11 The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a . [2]

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(b) Determine the nature of this stationary point. [2]

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(c) Find the equation of the curve.

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(d) Find the coordinates of any other stationary points on the curve.

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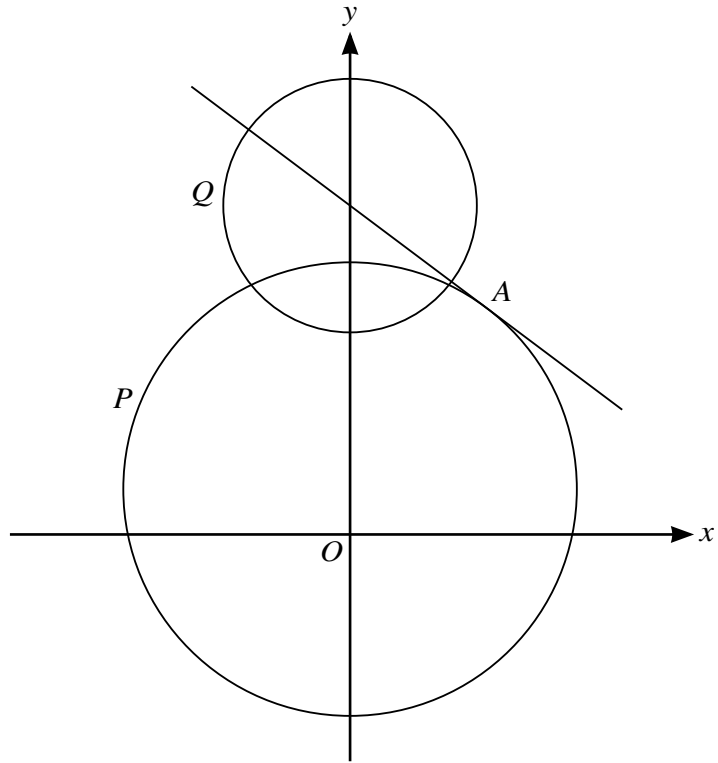
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The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y -axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P . [1]

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(b) Find the equation of the tangent to the circle P at A . [2]

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- (c) Find the equation of circle Q and hence verify that the y -coordinates of both of the points of intersection of the two circles are 11. [3]

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- (d) Find the coordinates of the points of intersection of the tangent and circle Q , giving the answers in surd form. [3]

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Additional Page

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